**P1.** Test the problems by: , where *n* = 1, 2, 4, 8 and *m* = 2:11

1. (by open NC rules, where *m =* 2:7)

2.

3.

4. (by open NC rules, where *m =* 2:7)

5.

+ Results:

Example = 1

Integral from 0 to pi of sin(x)/x use Open Rules

n = 1

m QNC(m) Error

2 1.9485571585149868 5.217e-02

3 1.8474907775521687 2.401e-03

4 1.8488738158196112 1.654e-03

5 1.8520199780790194 4.478e-05

6 1.8519953285262023 3.147e-05

7 1.8519360249149577 5.546e-07

n = 2

m QNC(m) Error

2 1.8742785792574932 1.206e-02

3 1.8516959584232047 1.302e-04

4 1.8517698706653425 9.027e-05

5 1.8519381372661514 5.860e-07

6 1.8519378175148116 4.134e-07

7 1.8519370487060038 1.769e-09

n = 4

m QNC(m) Error

2 1.8574232169529636 2.962e-03

3 1.8519224691786553 7.874e-06

4 1.8519269241813112 5.469e-06

5 1.8519370682602461 8.790e-09

6 1.8519370634747592 6.206e-09

7 1.8519370519702536 6.594e-12

n = 8

m QNC(m) Error

2 1.8533025791598587 7.374e-04

3 1.8519361478540937 4.882e-07

4 1.8519364238222962 3.392e-07

5 1.8519370522342653 1.360e-10

6 1.8519370521602783 9.601e-11

7 1.8519370519824196 2.506e-14

Example = 2

Integral from 0 to 2 of x^3 + 3\*x^2 + 6\*x + 9

n = 1

m QNC(m) Error

2 50.0000000000000000 1.905e-01

3 42.0000000000000000 0.000e+00

4 42.0000000000000000 0.000e+00

5 42.0000000000000070 1.692e-16

6 42.0000000000000070 1.692e-16

7 42.0000000000000000 0.000e+00

8 42.0000000000000000 0.000e+00

9 41.9999999999999930 1.692e-16

10 42.0000000000000000 0.000e+00

11 42.0000000000000070 1.692e-16

n = 2

m QNC(m) Error

2 44.0000000000000000 4.762e-02

3 42.0000000000000000 0.000e+00

4 42.0000000000000000 0.000e+00

5 42.0000000000000000 0.000e+00

6 42.0000000000000000 0.000e+00

7 41.9999999999999930 1.692e-16

8 42.0000000000000000 0.000e+00

9 42.0000000000000000 0.000e+00

10 42.0000000000000000 0.000e+00

11 42.0000000000000000 0.000e+00

n = 4

m QNC(m) Error

2 42.5000000000000000 1.190e-02

3 42.0000000000000000 0.000e+00

4 42.0000000000000000 0.000e+00

5 42.0000000000000000 0.000e+00

6 42.0000000000000000 0.000e+00

7 42.0000000000000000 0.000e+00

8 42.0000000000000000 0.000e+00

9 42.0000000000000000 0.000e+00

10 42.0000000000000000 0.000e+00

11 42.0000000000000000 0.000e+00

n = 8

m QNC(m) Error

2 42.1250000000000000 2.976e-03

3 42.0000000000000000 0.000e+00

4 42.0000000000000000 0.000e+00

5 42.0000000000000000 0.000e+00

6 42.0000000000000000 0.000e+00

7 42.0000000000000000 0.000e+00

8 42.0000000000000000 0.000e+00

9 42.0000000000000000 0.000e+00

10 42.0000000000000000 0.000e+00

11 42.0000000000000000 0.000e+00

Example = 3

Integral from 0 to 1 of exp(-2\*x)\*cos(3\*x)

n = 1

m QNC(m) Error

2 0.4330095425352287 1.421e+00

3 0.1616850223247737 9.605e-02

4 0.1711418874347122 4.318e-02

5 0.1787830975981431 4.632e-04

6 0.1788221886567740 2.447e-04

7 0.1788723575136770 3.581e-05

8 0.1788698658471496 2.188e-05

9 0.1788659121247597 2.241e-07

10 0.1788659263909217 1.443e-07

11 0.1788659517693205 2.426e-09

n = 2

m QNC(m) Error

2 0.2295161523773874 2.832e-01

3 0.1777144678935575 6.438e-03

4 0.1783548327189471 2.858e-03

5 0.1788666318054651 3.800e-06

6 0.1788663442051126 2.192e-06

7 0.1788659726116845 1.141e-07

8 0.1788659646986564 6.986e-08

9 0.1788659521484851 3.063e-10

10 0.1788659521681571 1.963e-10

11 0.1788659522032240 2.487e-13

n = 4

m QNC(m) Error

2 0.1906648890145150 6.597e-02

3 0.1787946215609708 3.988e-04

4 0.1788342663616330 1.771e-04

5 0.1788659691501592 9.475e-08

6 0.1788659617811400 5.355e-08

7 0.1788659522768798 4.115e-10

8 0.1788659522483885 2.523e-10

9 0.1788659522032134 3.083e-13

10 0.1788659522032332 1.975e-13

11 0.1788659522032686 6.207e-16

n = 8

m QNC(m) Error

2 0.1817621884243569 1.619e-02

3 0.1788615099258349 2.484e-05

4 0.1788639781416108 1.104e-05

5 0.1788659524909623 1.608e-09

6 0.1788659523654253 9.066e-10

7 0.1788659522035500 1.574e-12

8 0.1788659522034412 9.652e-13

9 0.1788659522032685 3.104e-16

10 0.1788659522032685 1.552e-16

11 0.1788659522032687 7.759e-16

Example = 4

Integral from 0 to pi/2 of sin(x)/x use Open Rules

n = 1

m QNC(m) Error

2 1.3995190528383288 2.098e-02

3 1.3703108833036850 3.292e-04

4 1.3704494407376344 2.281e-04

5 1.3707648513004667 1.957e-06

6 1.3707640602186739 1.380e-06

7 1.3707621580525458 7.370e-09

n = 2

m QNC(m) Error

2 1.3777658419596659 5.109e-03

3 1.3707351624818815 1.970e-05

4 1.3707434157061640 1.368e-05

5 1.3707622079844675 2.906e-08

6 1.3707621962728180 2.051e-08

7 1.3707621681171913 2.721e-11

n = 4

m QNC(m) Error

2 1.3725019517687655 1.269e-03

3 1.3707604981517116 1.218e-06

4 1.3707610079361172 8.464e-07

5 1.3707621687690719 4.484e-10

6 1.3707621685884799 3.166e-10

7 1.3707621681543454 1.043e-13

n = 8

m QNC(m) Error

2 1.3711964250104833 3.168e-04

3 1.3707620640549385 7.594e-08

4 1.3707620958232098 5.277e-08

5 1.3707621681640618 6.984e-12

6 1.3707621681612490 4.932e-12

7 1.3707621681544886 1.620e-16

Example = 5

Integral from 0 to 1 of exp(-x^2)

n = 1

m QNC(m) Error

2 0.6839397205857212 8.420e-02

3 0.7471804289095103 4.771e-04

4 0.7469923196130519 2.252e-04

5 0.7468337098497525 1.282e-05

6 0.7468293572005986 6.995e-06

7 0.7468237565709796 5.038e-07

8 0.7468239052669895 3.047e-07

9 0.7468241418413635 1.209e-08

10 0.7468241385434105 7.674e-09

11 0.7468241326412366 2.292e-10

n = 2

m QNC(m) Error

2 0.7313702518285631 2.069e-02

3 0.7468553797909873 4.184e-05

4 0.7468380575121313 1.865e-05

5 0.7468241699098985 4.967e-08

6 0.7468241534674009 2.766e-08

7 0.7468241322861899 7.046e-10

8 0.7468241324907157 4.308e-10

9 0.7468241328152192 3.739e-12

10 0.7468241328142127 2.391e-12

11 0.7468241328124166 1.397e-14

n = 4

m QNC(m) Error

2 0.7429840978003812 5.142e-03

3 0.7468261205274666 2.662e-06

4 0.7468250166550731 1.183e-06

5 0.7468241332296146 5.586e-10

6 0.7468241330465941 3.136e-10

7 0.7468241328106782 2.342e-12

8 0.7468241328113554 1.435e-12

9 0.7468241328124292 2.973e-15

10 0.7468241328124285 2.081e-15

11 0.7468241328124272 2.973e-16

n = 8

m QNC(m) Error

2 0.7458656148456951 1.283e-03

3 0.7468242574357302 1.669e-07

4 0.7468241882064696 7.417e-08

5 0.7468241328184021 8.001e-12

6 0.7468241328157892 4.502e-12

7 0.7468241328124203 8.920e-15

8 0.7468241328124230 5.352e-15

9 0.7468241328124269 1.487e-16

10 0.7468241328124271 1.487e-16

11 0.7468241328124271 1.487e-16

**P2.** Compares the m-point Newton-Cotes and Gauss-Legendre rules, where *m* = 2:6

1.

2.

3.

4. (by open NC rules, where *m =* 2:7)

5.

+ Results:

Example = 1

Approximating the integral from 0 to pi of sin(x)/x

m NC(m) GL(m) NCErr(m) GLErr(m)

--------------------------------------------------------------------------------------------------------------------------------

2 1.9485571585149868 1.8485715254386723 9.6620106532520733e-02 3.3655265437937398e-03

3 1.8474907775521687 1.8519760531665777 4.4462744302973256e-03 3.9001184111597809e-05

4 1.8488738158196112 1.8519367765282784 3.0632361628548654e-03 2.7545418768148977e-07

5 1.8520199780790194 1.8519370532954373 8.2926096553359585e-05 1.3129712872483879e-09

6 1.8519953285262023 1.8519370519779530 5.8276543736246467e-05 4.5130565951012613e-12

Example = 2

Approximating the integral from 0 to 2 of x^3 + 3\*x^2 + 6\*x + 9

m NC(m) GL(m) NCErr(m) GLErr(m)

------------------------------------------------------------------------------------------------------------------------------------

2 50.0000000000000000 42.0000000000000000 8.0000000000000000e+00 0.0000000000000000e+00

3 42.0000000000000000 42.0000000000001140 0.0000000000000000e+00 1.1368683772161603e-13

4 42.0000000000000000 42.0000000000000000 0.0000000000000000e+00 0.0000000000000000e+00

5 42.0000000000000070 41.9999999999998720 7.1054273576010019e-15 1.2789769243681803e-13

6 42.0000000000000070 42.0000000000000000 7.1054273576010019e-15 0.0000000000000000e+00

Example = 3

Approximating the integral from 0 to 1 of exp(-2\*x)\*cos(3\*x)

m NC(m) GL(m) NCErr(m) GLErr(m)

---------------------------------------------------------------------------------------------------------------------------------

2 0.4330095425352287 0.1902534783527516 2.5414359033196021e-01 1.1387526149483129e-02

3 0.1616850223247737 0.1789515407414870 1.7180929878494844e-02 8.5588538218467347e-05

4 0.1711418874347122 0.1788602886968944 7.7240647685563046e-03 5.6635063740706482e-06

5 0.1787830975981431 0.1788659790261257 8.2854605125443070e-05 2.6822857213115725e-08

6 0.1788221886567740 0.1788659524006470 4.3763546494518524e-05 1.9737844692002682e-10

Example = 4

Approximating the integral from 0 to pi/2 of sin(x)/x

m NC(m) GL(m) NCErr(m) GLErr(m)

---------------------------------------------------------------------------------------------------------------------------------

2 1.3995190528383288 1.3704190473439428 2.8756884683840411e-02 3.4312081054554966e-04

3 1.3703108833036850 1.3707634372201094 4.5128485080336844e-04 1.2690656210345708e-06

4 1.3704494407376344 1.3707621654290150 3.1272741685395289e-04 2.7254734025206062e-09

5 1.3707648513004667 1.3707621681583140 2.6831459782883371e-06 3.8256064982533644e-12

6 1.3707640602186739 1.3707621681544844 1.8920641855491027e-06 3.9968028886505635e-15

Example = 5

Approximating the integral from 0 to 1 of exp(-x^2)

m NC(m) GL(m) NCErr(m) GLErr(m)

---------------------------------------------------------------------------------------------------------------------------------

2 0.6839397205857212 0.7465946882828597 6.2884412226705821e-02 2.2944452956730199e-04

3 0.7471804289095103 0.7468145841912576 3.5629609708331955e-04 9.5486211694195688e-06

4 0.7469923196130519 0.7468244681309939 1.6818680062491431e-04 3.3531856691304540e-07

5 0.7468337098497525 0.7468241267662488 9.5770373255144037e-06 6.0461782247500651e-09

6 0.7468293572005986 0.7468241328901554 5.2243881716318441e-06 7.7728379288544147e-11